STUDY GROUP IN HISTORY OF MATHEMATICS – – - SOME HPM ACTIVITIES IN HONG KONG

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Abstract

In this paper the author shares with the readers the running of some local HPM activities of a study group in history of mathematics formed by mainly school teachers and curriculum officers of the Mathematics Section of the Education Bureau in Hong Kong.

Key words

History and pedagogy of mathematics, professional teacher development.

Περίληψη

Στο άρθρο αυτό ο συγγραφέας μοιράζεται με τους αναγνώστες την εξέλιξη ορισμένων δραστηριοτήτων που αφορούν στην Ιστορία και Διδακτική των Μαθηματικών μίας ομάδας μελέτης της Ιστορίας των Μαθηματικών που αποτελείτο κυρίως από καθηγητές του σχολείου και από μέλη της επιτροπής αναλυτικών προγραμμάτων του Γραφείου Εκπαίδευσης του Hong Kong.

Λέξεις κλειδιά

Ιστορία & Παιδαγωγική των Μαθηματικών, Επιμόρφωση εκπαιδευτικών.

1. Introduction

Thirty-five or forty years ago the topic of HPM (History and Pedagogy of Mathematics)[1] was a relatively new venture. With the hard work of many researchers and teachers in the intervening years this is no longer the case. For many years now various authors in different parts of the world have written on the important role played by the history of mathematics in mathematics education. The 10th ICMI Study conducted in 1998 focused on the role of the history of mathematics in the teaching and learning of mathematics, with its work reported in *History in Mathematics Education*: *The ICMI Study* (Fauvel & van Maanen, 2000). In this paper the author shares with the readers the

running of some local HPM activities of a study group in history of mathematics formed by mainly school teachers and curriculum officers of the Mathematics Section of the Education Bureau in Hong Kong.

2. General Framework

In a chapter of the book *The First Sourcebook on Asian Research in Mathematics Education: China, Korea, Singapore, Japan, Malaysia and India* a general framework of what the author has in mind concerning HPM activities is given. Generally speaking, teaching is to tell a story, a good story which arouses curiosity and excites imagination, a story about the long quest by the human mind for an understanding of the world around us. In this respect the history of mathematics is a particularly pertinent component in the task. The author continues to elaborate on this point (Siu, 2014):

"The history of mathematics is an academic discipline just like any other academic discipline, with its own scope of study, body of research, and literature. Although I do dabble in this area now and then, I consider myself an amateur in this academic discipline. I am not qualified as a historian of mathematics; at the most I am a friend of the history of mathematics. I am more interested in integrating the history of mathematics with the teaching and learning of mathematics. This is not the same as advocating the teaching of the history of mathematics in schools and universities. I do not think it proper to teach the history of mathematics *per se* in school. At the university this may constitute a course as an elective, but not a staple diet for mathematics students. However, I do advocate the integration, at all levels of mathematical study, of suitable material taken from the history of mathematics to enhance and enrich our teaching and to convey a sense of history.

The rationale behind my position is twofold. First of all, the basic tenet I hold is that mathematics is part of culture, not just a tool, no matter how useful this tool might prove to be. As such, the history of its development and its many relationships to other human endeavours from ancient to modern times should be part of the subject. Secondly, through my own experience in teaching and learning I have found that knowledge of the history of mathematics has helped me to gain a deeper understanding and to improve my teaching. Now, integrating the history of mathematics with teaching is only one of many ways to do this. Anything which makes students understand mathematics better and makes students get interested in mathematics may be a good way. The history of mathematics may not be the most effective choice, but I believe that, wielded appropriately, it can be an effective means. Moreover, knowledge of the history of mathematics may make a teacher "more patient, less dogmatic, more humane, less pedantic" and encourage a teacher to become "more reflective, more eager to learn and to teach with an intellectual commitment" (Siu, 1997/2000).

In coordinating a Topic Study Group (TSG17) on HPM activities at ICME-10 in 2004 at Copenhagen, Denmark together with Costas Tzanakis of the University of Crete, we pointed out that "despite its importance, history of mathematics is not to be regarded as a panacea to all pedagogical issues in mathematics education, just as mathematics, though important, is not the only subject worth studying," and further that "it is the harmony of mathematics with other intellectual and cultural pursuits that makes the subject even more worth studying" (Siu & Tzanakis, 2004). In this wider context, the history of mathematics has a yet more important role to play in providing a fuller education of a person, which is particularly pertinent in this age of mass education with the theme of "mathematics for all" (Siu, 1994).

I have likened the use of the history of mathematics in the classroom to an appetizer, a main course or a dessert, which caters respectively to motivation, content, or enrichment. Unlike the gastronomic analogue, a more fitting way is not to regard the use of the history of mathematics in the classroom in compartmentalized categories. In fact, it is even debatable whether the phrase "*using* the history of mathematics" should be employed at all. The word integrating, and better yet permeating, may be more appropriate.

Three aspects of the study of the history of mathematics are closely related and yet are separate issues: (1) doing research in the history of mathematics, (2) teaching the history of mathematics, and (3) integrating the history of mathematics with the teaching and learning of mathematics. HPM activities deal mainly with the third aspect, which can further be refined into three interrelated aspects: (3a) learning and teaching a certain subject area in mathematics, (3b) providing general motivation and enjoyment in studying mathematics, (3c) nurturing a deeper awareness of mathematics and its social and cultural context.

In terms of implementation there are four areas to note: (1) to consider epistemological issues relevant to the relations between mathematics, history, mathematics education and other disciplines; (2) to enrich teachers' education at all levels, both by introducing courses relating the history of mathematics to other disciplines and by letting teachers become acquainted with historically inspired material that can be or has been used in the classroom; (3) to construct and develop appropriate relevant didactical material, which can either be used directly in the classroom or constitute resource material for mathematics teachers; and (4) to present specific examples and the underlying rationale as an illustration of how history may contribute to the improvement of mathematics teaching by exciting the students' interest, enhancing their understanding of mathematical results or theories, or deepening their awareness of what mathematics really is (Siu & Tzanakis, 2004)."

3. Working with School Teachers

As a teacher of mathematics, the author wishes that students can be brought up in a classroom culture and environment that enables them to acquire active and effective learning habits so that they are able to access and read references; be able to write and speak clearly so as to communicate with others; be able to make sense out of mathematics and to explain what they comprehend; be willing to think, to query, to challenge, and to probe; have first-hand mathematical experiences so that they realize the dual natures of mathematics as an exact science as well as an imaginative endeavour, and as an abstract intellectual pursuit as well as a concrete subject with real-life applications; and appreciate the beauty, the import, the power, and the limitations of mathematics (Siu, 2014).

With school mathematics in mind the author tries to work with school teachers and to encourage them to participate in HPM activities. The author also offered a course titled "Development of Mathematical Ideas" at the University of Hong Kong from 1976 to 2005 when he retired (Siu, 1997/2000), at the same time collaborating with colleagues in the Mathematics Section of the Education Department of the Hong Kong Government[2], who can reach a wider network of school teachers in an official capacity, in conducting occasional seminars and workshops.

As an ineffective promoter working in an examination-oriented education environment the author had not done too well until rather recently. With the enthusiasm and able organization of a colleague in the EDB, Christine M. Y. Tang, and the dedicated effort of a very experienced school teacher, Jack C. K. Leung, a study group on the history of mathematics was formed in the summer of 2007 that meets about five times a year. From the summer of 2014 onward, owing to the posting of Christine Tang to another Section of the EDB, the Study Group continues to enjoy the able leadership of another young curriculum officer, Stanley C. Y. Lee. Members of the Study Group relish every single meeting in which they freely share ideas and experiences even though the Study Group is small with only a dozen stalwarts, whose dedication to their teaching profession the author much admires, knowing the very heavy workload and work pressure that local school teachers are placed under. Compared to school teachers elsewhere that the author has met in HPM conferences, this fledgling local group is just taking a small initial step and has a far way to go, but they are trying to move forward (Siu, 2014).

An exchange of emails between members of the Study Group after the first meeting in May of 2007 illustrates what the Study Group sets out to do. Two such emails from the initiator and the author are extracted below.

Dear All,

Thank you very much in participating in the first meeting of the Study Group on History of Math. I enjoy very much the sharing we had last Friday though we did not have enough time for an in-depth sharing. I do hope that we can build up this sharing culture, posing questions on the theme and trying to search for solutions of the problems posed in forthcoming meetings of the Study Group. Attached please find the powerpoint file that I presented last Friday, which I guess may be useful for those who could not attend our first meeting.

Below are the points that I can recall on the action forward of the Study Group (please do feel free to add what I miss out):

i) We agree that reading references/books/articles,... is very important to our development and we plan to have sharing of our teaching and learning experience on the theme and our reading experience in the meetings of the Study Group. The meeting will be arranged regularly, hopefully once every two months except very busy school months. (Attached please find the tentative plan of the Study Group)

ii) In between meetings, we can share experiences or pose questions on the theme through e-mails.

iii) The EMB will purchase reference books/CD-ROMs to facilitate the sharing. Attached please find the reference list. Please do add the references that you want us to purchase or you want to share with others.

iv) Mr Leung Chi-kit has produced some learning activities related to history of mathematics. EMB colleagues will reproduce these activities in CD-ROMs and will send to members for their references. Nevertheless, I would be grateful if you could keep these materials for your own uses and not for commercial purposes as we still need some time to polish the materials. If you have already designed some learning activities in the classrooms, please do share with us.

v) The second meeting of the Study Group is tentatively arranged to be held at 9:30 a.m. to 12:30 p.m. on 16 July (Mon). Each member is invited to prepare their sharing on their experiences in the theme (either reading experiences or experiences in incorporating history of math in the classroom).

vi) Feel free to invite other mathematics teachers interested in studying history of mathematics or incorporating history of mathematics in the classroom to join the Study Group. Just send me their names, the schools they work in and their contacts (for easy contacts, please send both phone numbers and e-mails) and tell them the date of our second meeting.

Thank you very much for your participation and looking forward to seeing you on 16 July.

Christine

28 May 2007

Dear Christine (and all friends in the Study Group),

Thank you for calling the meeting and sending us the material after the meeting.

> I wish this Study Group all success. More importantly, I hope all will find their time and effort well-spent and all will enjoy the experience, particularly a kind of 'esprit de corps'.
>
> Man Keung
>
> 29 May 2007

The main programme taken on by the Study Group in the initial stage is to study collectively two famous treatises, namely, Euclid's *Elements* and the ancient Chinese mathematical classics *Jiu Zhang Suan Shu* (九章算術 The Nine Chapters on the Mathematical Art). Members of the Study Group volunteer to present at regular meetings what they have studied from these two treatises, at times mixing with related topics from other sources in line with the objectives of HPM activities. These two primary source treatises are readily accessible. For instance, an English translation of *Elements* with copious commentaries is in the book by Sir Thomas L. Heath (Heath, 1925). An ancient edition of *Jiu Zhang Suan Shu* is in Volume 1 of the collection edited by GUO Shuchun (Guo, 1993), and is well supplemented by a translation into modern Chinese with commentaries by the same author (Guo, 2009). For English readers they can consult an English translation of *Jiu Zhang Suan Shu* together with the commentaries in the book by SHEN Kangshen, John Crossley and Anthony W. C. Lun (Shen & Crossley & Lun, 1999). Interested readers may also like to consult two more books on *Elements* by Robin Hartshorne (Hartshorne, 2000) and C. K. Leung (Leung, 2005). School teachers and even their pupils will enjoy and benefit from the second book if they are serious in studying mathematics and if they can read Chinese. The first book is written for a readership at a more advanced level, but the rich content is worth the effort.

The two famous treaties are chosen for study not only because of their significant roles in the history of mathematics as well as their rich content in mathematics, but also because a detailed study of these two treatises will enable readers to see the different approaches to mathematics that went on in the ancient world in the West and in the East. Broadly speaking it is commonly described as the "dialectic" approach and the "algorithmic" approach respectively, but this is perhaps an over-simplification, although the two approaches and the accompanying development indeed exhibited different features. Tradition holds that Western mathematics, developed by the ancient Greeks, is dialectic, while Eastern mathematics, developed by the ancient Egyptians, Babylonians, Chinese and Indians, is algorithmic. Even if it holds an element of truth as a broad statement, under more refined examination this thesis is an over-simplification. Borrowing the words adopted by Peter Henrici (Henrici, 1974) we can say that dialectic mathematics is a rigorously logical science, in which "statements are either true or false and objects with specified properties either do or do not exist". On the other hand, algorithmic mathematics is a tool for solving problems, in which "we are concerned not

only with the existence of a mathematical object but also with the credentials of its existence". In a plenary lecture given in Crete in July of 2002 the author attempted to synthesize the two aspects from a pedagogical viewpoint with examples from historical mathematical developments in Western and Eastern cultures (Siu, 2002). In the 19th ICMI Study Conference held in Taipei in May of 2009 the author reiterated this theme, focusing on proof, and discussed how the two aspects complement and supplement each other in proof activity. A procedural (algorithmic) approach helps to prepare more solid ground on which to build up conceptual understanding; conversely, better conceptual (dialectical) understanding enables one to handle algorithms with more facility, or even to devise improved or new algorithms. Like *Yin* and *Yang* in Chinese philosophy, these two aspects complement and supplement each other, each containing some part of the other (Siu, 2012).

4. Enrichment Seminars for School Teachers

In March of 2010 and in March of 2011 the hard work of members of the Study Group led to two seminars. The first seminar on "Symphony in trigonometry, Opus 360: Chords in harmony," and the second seminar on "Development of number systems," were both closely related to the local school curriculum, hence more than a hundred school teachers came each time. Finding that such kind of seminars received good reception by school teachers this activity gradually becomes an annual event for the Study Group. Since 2012 three more annual seminars had been held. The third one held in 2012 is on area and volume, the fourth one held in 2013 is on probability and the fifth one held in 2014 is on statistics. The sixth one to be held in May of 2015 will be on the topic of infinity. In working on these seminars, the Study Group becomes more strongly aware of how one should examine a topic from three perspectives: a historical perspective, a mathematical perspective, and a pedagogical perspective. Although the three are related, they are not the same; what happened in history may not be the most suitable way to go about teaching it, and what is best from a mathematical standpoint may not be so in the classroom and is almost always not the same as what happened in history. However, the three perspectives complement and supplement each other. For a teacher, it is good to know something about the historical perspective, to have a solid idea of the mathematical perspective, and to focus on the pedagogical perspective. To illustrate this point, let us say a bit more of the first three seminars.

The topic of the first seminar is a good example of a case where the teaching sequence *cannot* follow the path of historical development. Trigonometry arose from studying astronomy and calendar reckoning, which would be too complicated for school mathematics, as it involves spherical trigonometry. However, there are many intersections which make an integration of historical material in the teaching and learning of the subject possible. Trigonometry is also a topic which appears in different areas

(geometry, algebra, and calculus) and at different levels so that teachers may wish to know why and how the trigonometric functions seem to look so different in elementary mathematics and in advanced mathematics. Several intricately interwoven themes developed like those in a symphony, sometimes with variations. The seminar was given a playful title, not without a pun in mind! (The word "[half] chord" has an old meaning as the equivalent of the sine function. The word "harmony" has a meaning when trigonometric functions appear in a Fourier series.)

The seminar began with an account of the history of trigonometry from ancient to medieval times, not so much as a comprehensive and technical historical account, but as a sampling of the difficulties ancient peoples struggled with in solving trigonometric problems. Two useful references for this topic are: *The Mathematics of the Heavens and the Earth: The Early History of Trigonometry* (2009) by Glen van Brummelen and *Trigonometric Delights* (1989) by Eli Maor. This was followed by a discussion of the history of trigonometry in China (mostly of the transmission from European mathematics in the 17th and 18th centuries). To integrate these topics with classroom teaching, there was a workshop in making use of historical material in the teaching and learning of trigonometry. This workshop was conducted by several school teachers who shared their own classroom teaching experiences and their own worksheets.

The second seminar was motivated by a picture of the so-called *number tree* that commonly appears in a textbook, which offers a clear, but yet *too clear*, picture of the number system (besides, strictly speaking it is not a tree because the relationship involved is not entirely hierarchical). In the short introductory remarks to the seminar, the author tried to stress the complicated and intricate features of the development of number systems rather than the clear-cut and systematic impression one gathers from a standard number tree. This echoes our own learning experiences from kindergarten to university, in which the notions of different kinds of numbers proceed from vagueness to precision, just as was the case in history. This point was borne out by the remaining part of the seminar, which covered specific topics (integers, rational and irrational numbers, algebraic and transcendental numbers, and complex numbers) that were explained by several school teachers in the context of their classroom experience. The book *Numbers and Infinity: A Historical Account of Mathematical Concepts* (1981) by Ernst Sondheimer and Alan Rogerson is a useful reference, with a more advanced companion *Numbers* (1991, translated from the German original *Zahlen*, 2nd edition, 1988) by Hans-Dieter Ebbinghaus et al. In the concluding remarks of the seminar the author explained and illustrated the three aspects—historical, mathematical, and pedagogical—through three examples taken from the content of the seminar. One was "minus times minus is plus;" the second was "non-unique factorization and prime/irreducible element and ideal numbers;" and the third was "0.9999... equal to 1or not?"

The topic of the third seminar, which comes up in both junior and senior secondary school level or even in primary school level, serves to explain why a teacher would do well to know something about the two treatises *Elements* and *Jiu Zhang Suan Shu*. The seminar began with a talk on the treatment of calculation of area of rectilinear figures in ancient China, followed by the calculation of the area of a circle by the ancient Greeks, followed by the calculation of the volume of certain solids in ancient China. The second part of the seminar compared the calculation of the volume of a sphere in ancient China (*la* LIU Hui in the 3rd century and ZHU Geng in the 6th century) with that in ancient Greece (*la* Archimedes in the 3rd century B.E.C.). In the introductory and concluding remarks the author took the opportunity of discussing the topic of area and volume to explain the point put forth at the end of the preceding Section, namely, in what way the "dialectic" approach and the "algorithmic" approach complement and supplement each other. The seminar ended with an allusion to the modern concept of area and volume developed by Henri Lebesgue and Constantin Carathéodory in the early 1900s and the notion of dimension that was developed by a number of noted mathematicians of the early 20th century, with a brief mentioning of the fascinating topic of fractals, thereby indicating a continuation of the evolution of ideas from ancient to modern times.

5. Nothing Ventured, Nothing Gained

In a local workshop on HPM activities held for school teachers in 2005 the author gave a talk with the title "*Zhi y x ng n n* (knowing is easy and doing is difficult) or vice versa?" The original dictum summarizes a piece of Chinese wisdom that dates back to the ancient classics before our common era. More than two millennia later, Dr. Sun Yat-Sen, founder of the Chinese Republic in 1911, stressed the importance of a positive attitude towards action and deed over a passive attitude of wait-and-see by reversing the word order to *zhi n n x ng y* (knowing is difficult and doing is easy). The dictum was later extended by some eminent scholars to *zhi n n x ng n n* (both knowing and doing are difficult). No matter which of these one agrees with, nobody would be audacious enough to guarantee *zhi y x ng* y ; as the Western saying goes, "nothing ventured, nothing gained", or "there is no such thing as a free lunch"! To engage in HPM activities one has to invest time and effort to equip oneself for the task. There is no substitute for assiduous study on one's own. The author's experience is that knowledge is accumulated by bits and pieces over months and years and is never ending. It is no easy task, but it is meaningful and enjoyable. If we do not get involved or get started, then nothing will be accomplished (Siu, 2014). In an email written to members of the Study Group in April of 2008 the author elaborated on this point:

> Dear members of the Study Group,
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> Thanks to Christine for arranging a good meeting yesterday. Even though the number of participants was not as large as expected, we did have a fruitful

> discussion. Chi Kit, in his usual frankness which I respect and like, put his finger on the crucial question on participation and suggested that frequent absence of a member should be taken to mean a loss of interest in the study group or heavy commitment elsewhere (or both) so that in any case he or she can be left out in the mailing list. I take a more 'lenient' stand and see 'interest' in a wider context.
>
> There are various levels of interest. Some have a really deep interest, like that of Chi Kit (or of my humble self), so much so that they are willing to put in the time and effort to study relevant material, including primary sources (still not to the extent of a real professional historian of mathematics, of course). Some are building up this kind of interest. Some are interested in building up this kind of interest but do not know how to start. We should welcome everybody with that intent. We are all learning along the way. Anyone who expresses such intent, either in coming to meetings or in exchanging emails, is welcome to be an active member of the group. If a member loses interest in the study group or has heavy commitment elsewhere, then he or she would not mind being left out from the mailing list anyway.

Let me quote from an email I wrote to a colleague in the EDB on December 20, 2007:

> "Do not regard me as an expert. I am merely a mathematics teacher who happens to work along that line earlier than most. To a large extent I must admit that I have been very fortunate in that I could enjoy that kind of 'luxury' in spending my time and effort on this kind of endeavour when I joined the University of Hong Kong in the mid-1970s. Had I joined the tertiary community two decades later, I would have to spend my time on keeping my head above water (and to keep my job) by concentrating on some (small) part of technical mathematics. Experience in this kind of endeavour comes in bits and pieces, and knowledge is accumulated in months and years, and never ending. It is no easy task, but it is meaningful and enjoyable. All I know is that if we do not get involved or get started, then nothing would be accomplished. Hence, I would not be able to answer all queries; nobody can. But I am always ready to contribute whatever I am capable of doing. I look forward to the study group meeting on Friday."

As long as we feel happy (and at ease) with such meetings, I think it is worthwhile to keep it up. As I have always suggested, we can supplement with email exchanges in between. Time (and priority) is definitely a crucial factor. The HPM (History and Pedagogy of Mathematics) group in Taiwan, led by my friend Wann-Sheng (HORNG), has done very fruitful work. However, there is one factor our study group can hardly be matched up to them. Besides the dedication of members of that group, the group is built up and is formed by PhD and MA students who write their thesis on those topics. In this sense, they pursue their interest along with their daily research and study. For members of our group, the time and effort spent is "extra", so to speak. Perhaps I also quote from another email I wrote you on January 7, 2008 and on January 22, 2008:

> "Actually, I am thinking that perhaps the study group can contemplate building up some sort of website to accommodate material which members may find useful. We already have a batch of such material gathered so far, such as lists of

references, powerpoint files, work sheets, capsules (like those developed by Chi Kit), annotated syllabus prepared by Christine, etc. We can also edit the exchange of emails between some of us on the planning and the objectives of the study group. New material will be added as time goes on. When a new member joins the group, he or she will be given a link to that website. This would be one way to generate 'solidarity' of the group and to encourage discussion among members via email."

"The enthusiasm of the study group in the December meeting was not as high as before, probably because everybody is too much occupied with other things. Maybe the establishment of a 'chat room' can help to boost the mutual morale from time to time. This is what I suggested in my email dated January 7. Face-to-face meeting certainly has its merit. But when everybody is busy with what each has to take up in respective positions, assigning "homework" for the meeting can easily turn into a 'burden', which would further diminish the enthusiasm. I guess that is what Chi Kit means in maintaining that we should just go forward without assigning any goalpost but let things develop as we go forward. Hence, upkeeping the morale and affinity would be important. Christine has already provided strong support in building up a good library, collecting relevant material and distributing it to all, and in forming such a group. The question is how to maintain the enthusiasm and to convey the experience to a larger community. Keep going cheerfully! (Years ago there was no such group at all.)"

Hope to meet again in May or June. I will be out of town from April 13 to April 30, spending the two-and-half-week in Greece. Have a nice weekend.

Man Keung

April 4, 2008

Notes

1. The term HPM has become a shortened acronym for ISGRHPM (International Study Group on the Relations between the History and Pedagogy of Mathematics), which was established in 1976 as an affiliation of ICMI (International Commission on Mathematical Instruction). In this paper HPM is used in a broader sense to describe activities pertaining to the objectives and interests of the group, but not necessarily directly sponsored by the group. (For more detailed information about the group and its activities readers are referred to the official HPM website http://www.clab.edc.uoc.gr/hpm/.)
2. The Education Department was renamed as the Education and Manpower Bureau (EMB) in July of 1997 when the sovereignty of Hong Kong returned to China. It was renamed again in July of 2007 as the Education Bureau (EDB) after the protocol of manpower was moved to another Bureau.

References

Fauvel, J. & J. van Maanen (Eds.) (2000) *History in mathematics education: The ICMI study*. Dordrecht: Kluwer Academic Publishers.

Guo, S. C. (Ed.) (1993) *Collection of Chinese Classics in Science and Technology (Mathematics), Volume 1-5* [in Chinese 中國科學技術典籍通彙(數學卷)]. Zhengzhou: Henan Educational Press.

Guo, S. C. (2009). *Translation and Annotation of Jiuzhang Suanshu* [in Chinese 九章算術譯注]. Shanghai: Shanghai Antique Books.

Hartshorne, R. (2000). *Geometry: Euclid and Beyond*. Heidelberg-New York: Springer-Verlag.

Heath, T. L. (1925). *The Thirteen Books of Euclid's Elements*. 2nd edition, Cambridge: Cambridge University Press.

Henrici, P. (1974). Computational complex analysis, *Proc. Symp. Appl. Math*. 20, 79-86.

Leung, C. K. (2005). *A Guided Reading of Euclid's Elements* [in Chinese 幾何原本導讀]. Taipei: Chiu Chang Publishing.

Shen, K. S., J. N. Crossley &, A. W. C. Lun (1999). *The Nine Chapters on the Mathematical Art: Companion and Commentary*. Oxford: Oxford University Press.

Siu, M. K. (1994). My view on "Mathematics for all." [in Chinese 我看「大眾數學」]. In S. Yan, J. (Ed.), *Mathematics education in China for the 21st century* [in Chinese 面向21世紀的中國數學教育] (pp. 256-265). Nanjing: Jiangsu Educational Press.

Siu, M. K. (2000). The ABCD of using history of mathematics in the (undergraduate) classroom. (Original work published 1997). In V. Katz (Ed.), *Using History To Teach Mathematics: An International Perspective* (pp. 3-9). Washington, D.C.: Mathematical Association of America.

Siu, M. K. (2002). "Algorithmic Mathematics" and "Dialectic Mathematics": The *Yin* and *Yang* in mathematics education. In C. Tzanakis et al (Eds.), *Proceedings of the Second International Conference on the Teaching of Mathematics (at the Undergraduate Level)* (in CD-ROM). New York: John Wiley.

Siu, M. K. (2012). Proof in the Western and Eastern traditions: Implications for mathematics education. In G. Hanna, M.de Villiers (Eds.), *Proof and Proving in Mathematics Education: The 19th ICMI Study* (pp.431-440). New York-Heidelberg: Springer-Verlag.

Siu, M. K. (2014). "*Zhi yi xing nan* (knowing is easy and doing is difficult) or vice versa? —- A Chinese mathematician's observation on HPM (History and Pedagogy of Mathematics) activities, in *The First Sourcebook on Asian Research in Mathematics Education: China, Korea, Singapore, Japan, Malaysia and India,* edited by B.

Sriraman, Cai, J.F., K. Lee, L. Fan, Y. Shimuzu, C. Lim, K. Subramanian, Charlotte: Information Age Publishing.

Siu, M. K. & C. Tzanakis, (2004). History of mathematics in classroom teaching—Appetizer? Main course? Or dessert? *Mediterranean Journal for Research in Mathematics Education*, *3* (1-2), v-x.