FORWARD

Integrating History of Mathematics in Mathematics Education has been advocated since the second half of the 19th century, when great mathematicians like A. De Morgan, H. Poincaré, F. Klein and others explicitly supported this path and important historians like P. Tannery and later G. Loria showed an active interest on the role History of Mathematics can play in education. At the beginning of the 20th century, this interest was revived as a consequence of the discourse and the related debates on the foundations of mathematics. Later on, history became a resource for the various epistemological approaches, like Bachelard's historical epistemology, Piaget's genetic epistemology, and Freudenthal's phenomenological epistemology, at the same time stimulating the formulation of specific ideas and conclusions on the learning process.

This interest became stronger and more competitive in the period 1960-1980 in response to the "New Math" reform, when its proponents were strongly against "a historical conception of math education", whereas, for its critics, history appeared like a "therapy against dogmatism," conceiving mathematics not only as a language but also as a human activity. In 1969, the *National Council of the Teachers of Mathematics* (NCTM) in USA devoted its 31st Yearbook to the History of Mathematics as a teaching tool and in the 1970s a widespread international movement began to take shape, greatly stimulated and supported by the establishment of the *International Study Group on the Relations between History and Pedagogy of Mathematics* (the so-called *HPM Group*) affiliated to the *International Commission on Mathematical Instruction* (ICMI), originally in 1972 during the *2nd International Congress on Mathematical Education* (ICME 2) and finally in 1978[1]. Thus, during the last 40 years or so, integrating History of Mathematics in Mathematics Education has been developed as a worldwide intensively studied area of new pedagogical practices and specific research activities.

In this way, a gradually increasing awareness has emerged that since Mathematics is a human intellectual enterprise with a long history and a vivid present, mathematical knowledge is determined, not only by the circumstances in which it becomes a deductively structured theory, but also by the procedure that originally led or may lead to it, and which is indispensable for its understanding. Therefore, learning Mathematics includes not only the final "polished products" of mathematical activity, but also understanding implicit motivations, the sense-making actions and the reflective processes of mathematicians, which aim to the construction of meaning; that is, teaching mathematics should also give students the opportunity to "do mathematics". Therefore, perceiving Mathematics both as a logically structured collection of intellectual products and as processes of knowledge production, should be the core of the teaching of Mathematics. At the same time, it should also be central to the image of Mathematics communicated to the outside world.

Along these lines, putting emphasis on integrating historical and epistemological issues in Mathematics teaching and learning constitutes a possible natural way for exposing Mathematics in the making that may lead to a better understanding of specific parts of Mathematics and to a deeper awareness of what Mathematics as a whole really is. This is important for Mathematics Education, helping to realize that Mathematics is the result of contributions from many different cultures; has been in constant dialogue with other scientific disciplines, philosophy, the arts and technology; has undergone changes over time; has constituted a constant force for stimulating and supporting scientific, technical, artistic, and social development; and that throughout history there have been shifting views of what Mathematics is.

Aiming to increase the visibility of the *HPM* perspective within the Greek Educational Community, we have willingly accepted the invitation of the Editorial Board of *Education Sciences*[2], to act as guest editors of a special issue with focus on integrating *History and Epistemology in Mathematics Education*. This issue includes nine invited papers; five by Greek researchers and educators written in Greek and four by distinguished scholars in this area, written in English. Each text is accompanied by an abstract in both languages.

Among other things, the first three papers provide an overview of general ideas, conceptual frameworks and methodological schemes that have played an important role in shaping the theoretical background of introducing a historical dimension in Mathematics Education.

Yannis Thomaidis provides a well structured survey of the work done on introducing a historical perspective in Mathematics Education with emphasis on the case of Algebra.

Tinne Kjeldsen examines the possibility to establish an inquiry teaching and learning environment in Mathematics, through the History and Epistemology of Mathematics and provides relevant empirical data with Mathematics students and school teachers in Denmark.

After a concise outline of the meaning and content of a historical perspective in Mathematics Education, **Man Keung Siu** describes and analyses related activities of a group of school math teachers and educators in Hong Kong, with emphasis on a comparative study of the development of Mathematics in the Western and Eastern civilizations.

The remaining five papers discuss specific issues in this domain and are ordered according to the instructional level; from elementary school to the university.

Mathaios Anastasiadis and **Costas Nikolantonakis** provide information on an experimental teaching approach of isoperimetric problems and the relation of perimeter to area, with 6th-grade students, where excerpts from Pappus and Polybius' texts have been used, appropriately adapted to the students' educational and cognitive level.

Evangellos Panagiotou gives an interesting account of the historical development of

logarithmic concepts and presents a didactical sequence based on this development that has been implemented in upper high school (11th grade).

Adriano Demattè and **Fulvia Furinghetti** examine the advantages and difficulties of the *hermeneutic approach* as a way to implement history in mathematics teaching. In this context they provide results of an experimental teaching on the function concept with last-year high school students in Italy with a human sciences orientation. More specifically, it concerns the exponential and logarithmic functions, using excerpts from Euler's *Introductio in analysin infinitorum*.

These two papers consider the same mathematical subject from different perspectives, thus nicely complementing each other.

Michael Kourkoulos and **Constantinos Tzanakis** analyze their experimental teaching of prospective elementary school teachers on using statistical data in the context of the philosophical debate on the existence of human free will and its limitations imposed by social conditions and factors. To this end, texts of the 19th century Belgian pioneer statistician A. Quetelet have been used, that reveal the didactically beneficial interrelation between statistics and philosophy.

Uffe Jankvist's paper is informative and rich in ideas on the theoretical background of a historically oriented approach to the teaching of Mathematics, which touches upon philosophical issues on both the nature of Mathematics itself and its applications. The paper informs about the author's empirical research to identify long-term effects that an exposure to primary historical sources may have on mathematics students, in the context of a teaching module on *Graph Theory*.

Finally, **Theodorus Paschos** describes a didactical experiment with first-year mathematics students on the *Fundamental Theorem of Calculus*, based on original texts by Galileo and his late Middle Age predecessors, like W. Heytesbury, N. Oresme etc.

For the interested reader's convenience and further orientation, a concise bibliography, mainly of collective works, is provided at the end of the Greek version of the present introduction (see pp.4-6).

Michael Kourkoulos

Constantinos Tzanakis

Σημειώσεις

1. *Historia Mathematica* 5, 1978, 76.

2. *Education Sciences* (in Greek: "*Επιστήμες Αγωγής*") is the oldest Greek scientific journal in the domain of the Science of Education. For the last 14 years it has been published by the *Faculty of Education* of the University of Crete.